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ORIGINAL RESEARCH

Developing an economical model for reliability allocation of an electro-optical system by considering reliability improvement difficulty, criticality, and subsystems dependency

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Abstract

The nature of electro-optical equipment in various industries and the pursuit of the goal of reducing costs demand high reliability on the part of electro-optical systems. In this respect, reliability improvement could be addressed through a reliability allocation problem. Subsystem reliability must be increased such that the requirements as well as defined requisite functions are ensured in accordance with the designers' opinion. This study is an attempt to develop a multi-objective model by maximizing system reliability and minimizing costs in order to investigate design phase costs as well as production phase costs. To investigate reliability improvement feasibility in the design phase, effective feasibility factors in the system are used and the sigma level index is incorporated in the production phase as the reliability improvement difficulty factor. Thus, subsystem reliability improvement priorities are taken into consideration. Subsystem dependency degree is investigated through the design structure matrix and incorporated into the model's limitation together with modified criticality. The primary model is converted into a single-objective model through goal programming. This model is implemented on electro-optical systems, and the results are analyzed. In this method, reliability allocation follows two steps. First, based on the allocation weights, a range is determined for the reliability of subsystems. Afterward, improvement is initiated based upon the costs and priorities of subsystem reliability improvement.

Keywords Reliability allocation · Design and production cost · Reliability improvement difficulty · Sigma level · Design structure matrix (DSM)

List of symbols

R_{sys}	Whole reliability of system
R_{goal}	Goal reliability
R_i	Reliability of subsystem
λ_{goal}	Goal failure rate
λ_i	Failure rate of subsystem
R_{min}	Minimum reliability
R_{max}	Maximum reliability
C_1	Design phase cost of system
C_2	Production phase cost of system
N	Number of subsystems
A_i	Production cost of subsystem
$B1_i$	Reliability improvement cost of subsystem in the design phase
$B2_i$	Reliability improvement cost of subsystem in the production phase
u_1	Predetermined budget for improvement in the design phase
u_1	Predetermined budget for improvement in the production phase
i	Interest rate
T	Duration of the project for design and product development
F_{d_i}	Feasibility factor of increasing the reliability for subsystem in the design phase
F_{p_i}	Difficulty factor of increasing the reliability for subsystem in the production phase
W_i	Allocation weight (criticality and dependency factors)

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Introduction

Product reliability evaluation follows product design process and is regarded as an inextricable part of this process (Liu et al. 2014). In different industries, the reliability optimization problem can be addressed by regarding a product as a system comprising a number of subsystems. This problem is stated using system structure and limitations as well as the characteristics and arrangement of subsystems and components. In fact, reliability improvement refers to the enhancement of reliability in such a way that the functions required by the system are ensured (Falcone et al. 2014). Reliability improvement problems include three procedures: (1) increasing subsystem reliability (reliability allocation), (2) use of redundant components in parallel (redundancy allocation problem), and (3) application of the two aforementioned procedures (reliability-redundancy allocation problem) (Mellal and Zio 2016). In these problems, the objective is to maximize reliability in the face of cost, weight, and volume limitations. Reliability allocation has an essential link with reliability design and serves as an important activity in product design and development. Hence, it is imperative to evaluate system behavior, function, and parameters by using failure effects and data, subsystem dependency, and the degree of reliability improvement. In fact, to determine subsystem reliability based on goal reliability, attention must be paid to improvement opportunities and priorities based on the real potential of reliability improvement (Yadav and Zhuang 2014). Besides, investigation of the relationship among subsystems and their importance links failure analysis process and reliability improvement feasibility to the product design and development process (Zhuang et al. 2014).

System reliability influences all system costs during the system's life cycle. In order to reduce these costs, it is imperative to take account of various costs in reliability allocation and to reduce those costs by improving reliability in the design phase (Nguyen and Murthy 1988).

Reliability problems mostly focus on the design process. The most important system problems in the design process are the corollary of changes in production and assembly. Failure to control variability in the production process leads to increased costs and diminished system reliability. Hence, the production phase is crucial for system reliability and uncertainties existing in this stage and leads to the failure of the designed reliability required (Karaulova et al. 2012).

The reliability of the production process is a function of the capability of the critical variables of the process, complexity of activities, efficiency of time, and costs and capability of equipment and workforce to perform the defined missions without failure (Karaulova et al. 2012; Ávila 2015). Reliability estimation based on production phase data is an effective instrument to investigate the effect of production process on system reliability (Jinghuan et al. 2012). Variability of production processes and system performance can be quantitatively estimated, and the control and reliability improvement in systems can be investigated using process capability indexes (PCI) (Pearn * et al. 2005; Baril et al. 2011), which reflect the condition of reliability and quality in the production process (Jeang and Chung 2008). There are papers that they have investigated the performance of production process by using PCI like Cp, Cpk, sigma level and studied the product design based on the six sigma and reliability. Process performance evaluation using qualitative data is often based on the number of defective items. Used in this procedure is the sigma level index, to which reliability can be directly attributed. For example, for a sigma level of 3, the probability of the system's performing a defined function within a day under existing conditions is 99.73% and the probability of failure is 0.27% and for achieving 100% reliability, it is unnecessary to optimize in 6 sigma level (Baril et al. 2011).

There are several approaches for determining reliability allocation weights. In, these methods, the aim is to blend several factors to compute allocation weights. Afterward, considering the goal reliability, the reliability of subsystems or components is allocated. The traditional methods include aeronautical radio, incorporated (ARINC), advisory group in reliability of electronic equipment (AGREE), feasibility of objective (FOO) and averaged weighted method. Wang et al. (2001) investigated seven comprehensive factors: failure frequency, criticality, maintainability, complexity, production technology, work condition, and cost. The relationship between these factors and reliability was investigated, and allocation weights were computed accordingly. In the study by Jafarsalehi (2009), in order to conduct an accurate functional computation of subsystem reliability, factors are divided into subfactors. In this method, key parameters are defined and converted to understandable and proper subfactors. All of the subfactors are defined quantitatively and normalized. Then, normalized factors are summed and normalized again. This obtained weight is used for calculation of subsystem reliability. This is especially appropriate for stages lacking sufficient data regarding the factors or sufficient expert experience. Chang et al. (2009) used the maximum entropy-ordered weighted average (ME-OWA) method to allocate the weights, in which the optimal weighted vector is determined under maximum disorder and the defects of the FOO method are modified. In addition to using ME-OWA method, Liaw et al. (2011) investigated the indirect relationships between the subsystems using the decision-making trial and evaluation

(DEMATEL) method. These two studies employed the factors of the FOO method. Falcone et al. (2014) conducted reliability allocation by taking account of criticality, complexity, functions, and effectiveness factors and selected the most critical subsystem. Afterward, they evaluated the problem of redesigning the critical subsystem or adding a parallel subsystem in order to improve system reliability by investigating costs, risks, time, and the degree of improvement achieved. In order to take into consideration the vague and unclear priorities of expert judgment, Chen et al. (2015) added minimum variance to ME-OWA model and created the maximum entropy minimum variance-ordered weighted averaged (MEMV-OWA) model to compute the weight of the effective factor. Finally, using the analytical hierarchy process (AHP), the allocated weights were computed for automobile power transmission systems. Di Bona et al. (2016) investigated the subsystems and the factors through an AHP problem for a spatial sample. The factors considered include the degree of criticality, complexity, function, effectiveness index, technology, and electronic performance index. If W_i denotes allocation weight, subsystem reliability is determined through Eq. 1:

$$R_i = \left(R_{\text{goal}}\right)^{W_i} \text{ or } \lambda_i = W_i \times \lambda_{\text{goal}} \quad (1)$$

In a collection of papers including Yadav et al. (2006), Yadav (2007), and Itabashi-Campbell and Yadav (2009), allocation weights are linked to failure modes and effects analysis (FMEA) using risk priority numbers (RPN), where factors are multiplied by each other to compute RPN. In these papers, linear scaling is assumed and reliability is not allocated in accordance with criticality or potential for improvement. Kim et al. (2013) put forth an approach for computing weights, where exponential transformation function is used in place of the ordinal 10-item rate to compute failure severity, and where the exponential relationship between failure severity and failure effect is taken into consideration. The method focuses more intensely on subsystems with higher failure severity. Yadav and Zhuang (2014) considered weighted allocation for the improvement level of failure rate in series systems, in such a way that the degree of modified criticality is computed by considering a nonlinear relationship between failure severity and failure effect and between efforts for improvement and failure rate. In the paper by Zhuang et al. (2014), the degree of modified criticality is blended with functional dependency and based on the degree of significance of either, the allocation weight is computed so as to determine subsystem reliability. In this study, difficulty and complexity factors are set equal to 1.

In some papers, reliability allocation is carried out using a mathematical programming model. In the study by Mettas (2000), a model was developed to minimize design costs under reliability limitations and the reliability improvement feasibility factor. This factor was initialized according to expert judgment. The model is presented by Eq. 2:

$$\begin{aligned} \min C = \sum_{i=1}^{N} e^{\left[(1-f_i)\left(\frac{R_i - R_{i,\min}}{R_{i,\max} - R_i}\right)\right]} \\ s.t.\ R_{\text{system}} \geq R_{\text{goal}} \\ R_{i,\min} \leq R_i \leq R_{i,\max} \\ 0 \leq R_i \leq 1 \end{aligned} \quad (2)$$

In the study by Kumral (2005), the reliability variance of subsystems was added to the objective function of Mettas (2000) model. This model is solved by using the genetic algorithm designed for mining production systems, and the feasibility factor is initialized according to expert judgment. After incorporating failure rate and costs into the objective function of Mettas (2000) model, Zhang et al. (2007) defined an index known as subsystem importance, which is determined by using cost function derivative in relation to failure rate. Three ranges are defined for the values of importance, based on which three measures of reliability reduction, unimportant component, and reliability improvement are carried out. In the paper by Farsi and Jahromi (2012), effective feasibility factors such as complexity, criticality, state of art, operational profile, and availability were used for the reliability improvement feasibility factor, and the model was solved for a complex spatial system by using the genetic algorithm. Liu et al. (2014) added to the problem a new factor named manufacturing consistency, which is also known as PCI and is measured using the Cpk index. This index (as a variable) together with its costs is added to the model and is solved using the genetic algorithm after converting the primary model to the MDO model.

In the paper by Chen et al. (2013), optimization of the system is investigated by considering failure dependence and it is studied in K-out-of-n redundant systems in order to increase reliability by Mortazavi et al. (2016, 2017).

Investigation of previous research yields the conclusion that the investigated feasibility factors are considered as allocation weights. In this method, higher reliability is allocated to the subsystem with lower weight (higher reliability). Hence, these factors are not incorporated in the identification of the factors of subsystem reliability improvement priorities. Another point worth mentioning is the exclusion of system performance and costs from the production phase in allocation problems. Moreover, some studies have taken the intensity of subsystem interrelations into consideration, while they have ignored the type of relationships and their significance, despite the role of this issue in improving system design and modification.

The present study is an attempt to develop a model for subsystem reliability allocation aimed at maximizing system reliability and minimizing costs, such that reliability improvement priorities, criticality factors, and degree of subsystem interdependency are taken into consideration. In view of the importance of the reliability of the production phase, this study investigates design phase costs as well as production phase costs. For two reasons, these costs are investigated separately as two objective functions in the model: First, the amount of design and production cost is unequal and the smaller cost should not be affected by the larger cost. Second, the importance degree of costs is different in any system and project, and by separating design and production cost, the priorities of costs can be examined. In order to consider the priority of subsystem reliability improvement, the factor of reliability improvement difficulty is investigated. In this connection, the effective feasibility factors in the system are used for the factor of reliability improvement in the design phase, and the MEMV-OWA method is used to compute the weight of these factors. Besides, the sigma level index is computed so as to evaluate subsystem performance in the production phase and is incorporated into production cost function as a reliability improvement difficulty factor. In this study, another factor known as subsystem dependency was investigated using design structure matrix (DSM). This factor is incorporated into the model's limitations together with modified criticality. Finally, the multi-objective model is converted to a single-objective model using goal programming (GP), and reliability allocation for the electro-optical system is investigated. The proposed model provides greater flexibility for design engineers who seek to regulate reliability objectives in accordance with reliability improvement difficulty, degree of criticality, and subsystem dependency.

The remainder of this paper is structured as follows: In "Statement of the problem" section, the main problem is defined, and the proposed model as well as the relationships pertaining to costs and each factor is presented. In "Goal programming" section, the GP model is presented. In "Practical example" section, the proposed model is solved for the electro-optical system and the results are analyzed. Finally, in "Conclusion" section, general conclusions and suggestions for further research are presented.

Statement of the problem

After analysis of various reliability allocation models, in this paper, reliability improvement in the subsystems of electro-optical systems are investigated through a multi-objective optimization model. This model is developed for series systems with subsystems with exponential failure rates. Besides, it is assumed that subsystem failures are independent, and that systems and subsystems only have two function states, i.e., work and failed. Figure 1 presents the proposed method.

Proposed model

$$\max R = R_{\text{system}} \tag{3}$$

$$\min C_1 = \sum_{i=1}^{N} B1_i \cdot \left(e^{\left(1-F_{\mathrm{d}_i}\right)\left(\frac{R_i - R_{\min}^{w_i}}{R_{\max}^{w_i} - R_i}\right)} - 1 \right) \tag{4}$$

$$\min C_2 = \sum_{i=1}^{N} A_i + B2_i . \left(R_i^{F_{\mathrm{p}_i}} - R_{\min}^{w_i \times F_{\mathrm{p}_i}} \right) (P/A, i, T) \tag{5}$$

s.t.

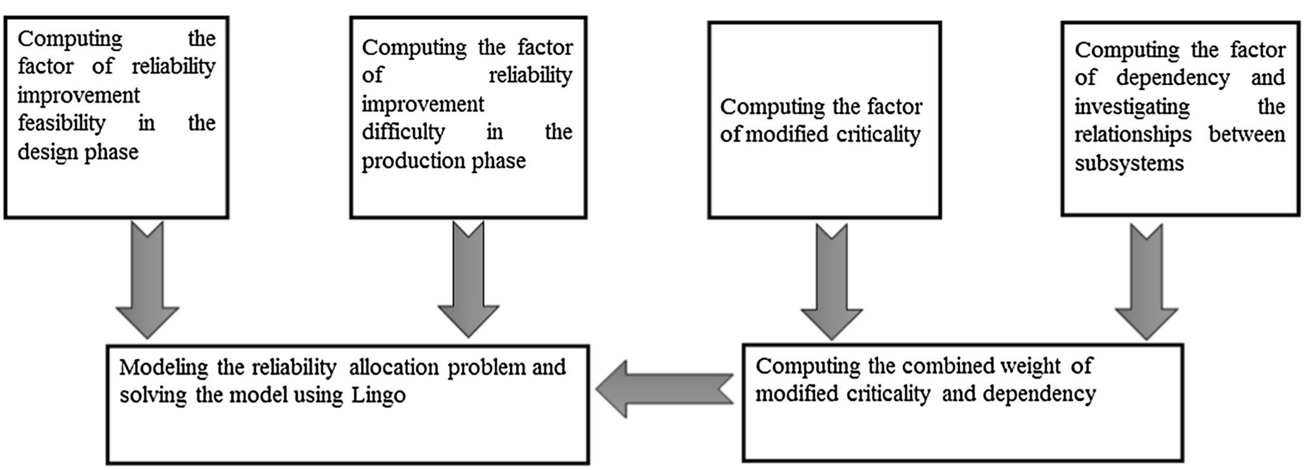

Fig. 1 Steps of proposed method

$$R_{\text{system}} \geq R_{\text{goal}} \tag{6}$$

$$R_{\min}^{W_i} \leq R_i \leq R_{\max}^{W_i} \tag{7}$$

$$0 \leq R_i \leq 1 \tag{8}$$

Design phase cost

Design cost relates to research and development cost for improving reliability and depends upon factors such as the time of development and redesign, efforts, etc. (Huang et al. 2007). This is extra design cost which improves reliability in comparison with the minimum amount. Subsystem design cost can be computed by analyzing previous data or similar distinct data according to Eq. 9. According to this definition, reliability improvement is feasible before the costs approach infinity (Öner et al. 2010).

$$C_1 = \sum_{i=1}^{N} B_1 \cdot \left(e^{(1-f_i)\left(\frac{R_i - R_{i,\min}}{R_{i,\max} - R_i}\right)} - 1 \right) \tag{9}$$

According to Fig. 2, this equation is an exponential function between reliability and design costs. Increase in reliability from 70 to 75% is less difficult than from 90 to 95%, which is due to increased costs and difficulty of reliability improvement for systems with higher reliability (Mettas 2000).

Reliability improvement feasibility factor in design phase

f_i denotes reliability improvement feasibility. Lower values of this factor represent greater reliability improvement difficulty and demonstrate the cases where subsystems enjoy higher reliability. As illustrated in Fig. 2, in this case, the cost function approaches infinity sooner (Mettas 2000). Most studies have assigned a value between 0 and 1 to this factor in accordance with expert judgment. In this paper, this factor is calculated by considering the features and behavior of the system and the opinion of the experts.

This factor can be computed by investigating feasibility factors in the system, including complexity, technology, operational time, operational profile, safety, reparability, availability, subsystem importance, and criticality. As mentioned in Refs. Wang et al. (2001) and Falcone et al. (2014), the feasibility factors must be functional and appropriate for the system in question. In this paper, using questionnaire and expert judgment, the factors affecting the system in question are selected. Based on the relationship existing between selected factor and reliability, in each subsystem, the value of φ_i is considered between 1 and 10 for each factor (Chang et al. 2009; Liaw et al. 2011). In the case of multiple experts, the average or mode of opinion is computed. The weight of selected factors is computed using MEMV-OWA model presented in Eq. 10, too (Chen et al. 2015):

$$\begin{aligned} &\max: -\sum_{i=1}^{n} w_i \ln w_i \\ &\min: \frac{1}{n}\sum_{i=1}^{n} w_i^2 - \frac{1}{n^2} \\ s.t.\quad &\sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, (0 \leq \alpha \leq 1) \\ &\sum_{i=1}^{n} w_i = 1, 0 \leq w_i \leq 1, i = 1, 2, \ldots, n \end{aligned} \tag{10}$$

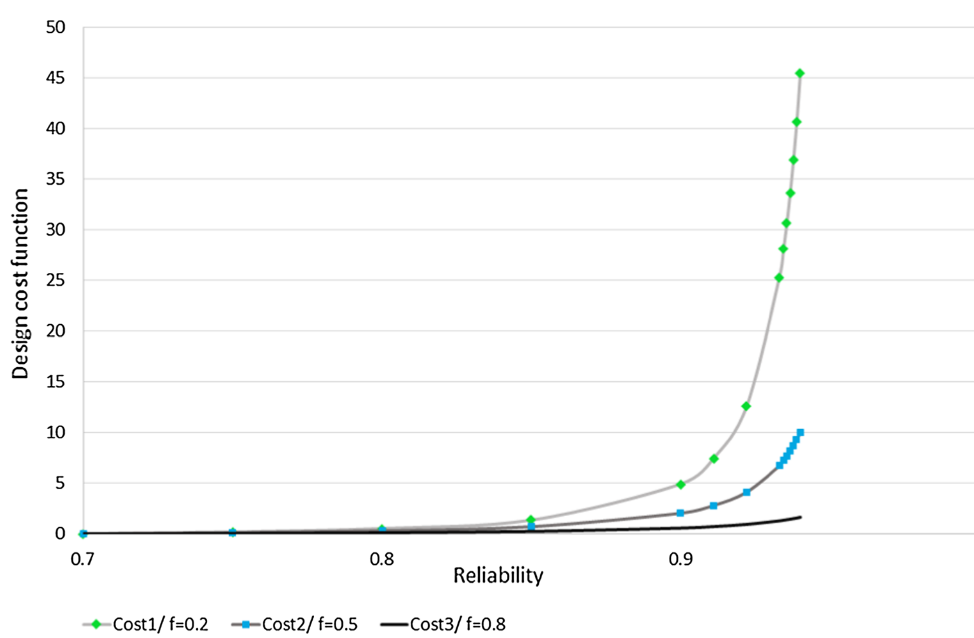

Fig. 2 Design cost function versus reliability and effect of feasibility factor

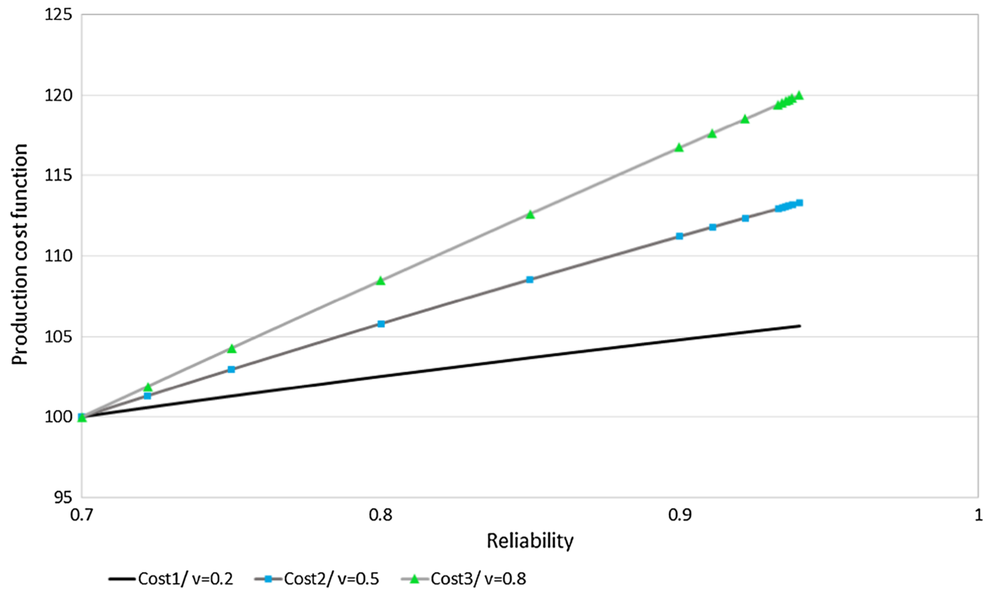

Fig. 3 Production cost function versus reliability and effect of difficulty factor

n is the number of factors and α is the degree of integration and position of OWA operator which adopts a value between 0 and 1. Values closer to 0 indicate pessimistic opinion and those closer to 1 indicate optimistic opinion of the expert. The models shows that non-certain data of decision-makers' experience continue to be optimized as much as possible by maximizing entropy. Besides, minimizing the variance of the weighted vector is a potential way to prevent the overrating of decision-maker priorities (Chen et al. 2015). If w_i represents the feasibility factors weight obtained by solving the MEMV-OWA model and φ_i represents the values of each factor in each subsystem, the ultimate weight of feasibility of the design is determined by Eq. 11. In this case, the priorities of subsystem reliability improvement are investigated in a functional fashion.

$$fd_i = \sum_{i=1}^{n} w_i \times \varphi_i$$
$$F_{d_i} = \frac{fd_i}{\sqrt{\sum_{i=1}^{N} fd_i^2}} \tag{11}$$

Production phase costs

The reliability of the production process is a key issue which ensures the stability and sustainability of the production system operations, increases quality, and decreases production loss. Being well informed of the reliability of the production process and using this information is crucial for identification and management of failures (Karaulova et al. 2012). Producing a product with high reliability requires the use of new material and technology as well as advanced production process (Huang et al. 2007). Production costs include the costs of production and installation of subsystems in the system. According to Eq. 12, production costs for minimum reliability are a basic and reference cost (Öner et al. 2010).

$$C_2 = \sum_{i=1}^{N} A_i + B_{2_i} \cdot \left(R_i^v - R_{i,\min}^v\right) \tag{12}$$

This production cost is an extra cost which moves reliability from $R_{\min}$ to R_i. An increase in reliability leads to an increase in production costs, forming a linear relationship as demonstrated in Fig. 3 (Jin and Wang 2012).

Generally, an increase in reliability is accompanied by an increase in the related costs, as demonstrated in Figs. 2 and 3. Therefore, in reliability optimization problems, the objective is to maximize system reliability and minimize costs. In Eq. 5, production cost is converted to present value by using engineering economic equations until design and production cost can be compared.

Reliability improvement difficulty factor in production phase

In Eq. 12, v represents the reliability improvement difficulty factor, an increase in which leads to an increase in reliability improvement difficulty. That is, the subsystem in question has a better performance (lower variability) in the production phase, making it more costly to improve its reliability. As demonstrated in Fig. 3, in this case, the costs approach infinity sooner. Most studies have assigned a value between 0 and 1 to this factor according to expert judgment.

In the present paper, using the sigma level index, the system performance in the production phase is investigated, and this index is considered as the factor of reliability improvement difficulty in the production cost function. Assuming that d_i denotes the number of defects in each subsystem, u_i stands for the investigated units, and o_i represents the number of failure chances. DPO_i is computed through Eq. 13, and F_{p_i} (which is the sigma level index) is determined through the statistical tables of standard distribution. After descaling F_{p_i} through the Euclidean norm method, this figure is incorporated into the model. Thus, reliability improvement priorities in the production phase are investigated.

$$\text{DPO}_i = \frac{d_i}{u_i \times o_i} \tag{13}$$

Modified criticality factor

Use of failure analysis data during the FMEA process in reliability allocation helps to demonstrate a realistic picture of the system's behavior and performance. The paper by Yadav and Zhuang (2014) discusses the notion that increased subsystem failure rate is accompanied by diminished effort for reliability improvement, and that there is a nonlinear relationship between these two phenomena. Thus, the potential and priorities of reliability improvement are demonstrated and the improved failure rate is computed. Besides, in this method, a factor known as the degree of difficulty and complexity is taken into consideration and initialized in such a way that the improved failure rate does not become negative. Higher degrees of subsystem criticality require higher reliability. Criticality weight is computed through the following equations:

$$\begin{aligned} \bar{S}_{ij} &= \exp(\alpha S_{ij}) \\ \bar{S}_i &= \max(\bar{S}_{i1}, \bar{S}_{i2}, \ldots, \bar{S}_{ij}) \end{aligned} \tag{14}$$

In Eq. 14, S_{ij} represents the failure severity of subsystem i at failure mode j. $\bar{S}_i$ is normalized and substituted into Eq. 17.

$$E_i = \frac{-\ln \lambda_i}{r} \tag{15}$$

In Eq. 15, r stands for the decrease rate of λ and E_i represents efforts for reliability improvement. λ_i is the summation of failure rate of various failure modes which is computed through Eq. 16.

$$\begin{aligned} \lambda_i &= \sum_{j=1}^{m} \lambda_{ij}, \\ \lambda_{ij} &= \exp(-9.99 + 0.7702 O_{ij}) \end{aligned} \tag{16}$$

O_{ij} stands for failure occurrence probability for subsystem i at failure mode j. E_i is normalized and substituted into Eq. 17, and the final value of criticality is computed.

$$\begin{aligned} c_i &= 1 - \left(\frac{s_i/e_i}{\sum_{i=1}^{N} s_i/e_i} \right) \\ C_i &= \frac{c_i}{\sum_{i=1}^{N} c_i} \end{aligned} \tag{17}$$

Dependency factor

The design structure matrix is a network modeling instrument used in system analysis and integration. It is used to demonstrate the components of a system, the interactions between subsystems or their components, and the power of interactions. DSM is a square matrix with identical titles for rows and columns, in which a sign out of the main diagonal demonstrates the interrelation and interdependency between one subsystem and others. Movement along a row shows the subsystem through which the subsystem input positioned on that row passes, and movement along a column shows all the subsystem output (Eppinger and Browning 2012). Sosa et al. (2003) investigated the type of relationships between subsystems that brief definition is presented in Table 1.

Subsystem dependency is identified using interviews with experts and sometimes using engineering documentation. The intensity and importance of the relationships are determined using Table 2, (Helmer et al. 2010).

When the objective is to modify and improve the system, the interdependency and the relationships between subsystems must be investigated. In this study, using paired comparison matrices, the weight of each relationship is determined, and, by multiplying each relationship weight

Table 1 Kinds of current relationships between subsystems or components of system

Interaction	Definition
Spatial	Functional requirement for alignment, orientation, serviceability, assembly
Structural	Functional requirement for transferring design loads, forces
Energy	Functional requirement for transferring heat, vibration, and electrical energy
Material	Functional requirement for transferring air, oil, fuel, or water
Information	Functional requirement for transferring signals or controls

Table 2 Intensity of existing relationships in the DSM

+ 2	Relationship is necessary for functionality
+ 1	Relationship is beneficial for functionality
0	Relationships don't affect functionality

by the intensity of the existing relationship and computing their summation for each cell, the various relationships exiting in each cell are simplified. Afterward, D_i, which stands for the weight of dependency of each subsystem on other subsystems, is computed. This weight is computed through Eq. 18, where d_{ir} represents the relationships within each column and d_{is} represents the relationships within each row. Subsystems with higher dependency, i.e., variation, result in greater variation in system design, which can affect general system reliability.

$$d_i = \sum_{\substack{r=1 \\ r \neq i}}^{N} d_{ir} + \sum_{\substack{s=1 \\ s \neq i}}^{N} d_{is} \qquad D_i = \frac{d_i}{\sum_{i=1}^{N} d_i} \tag{18}$$

In what follows, the two factors D_i and C_i are blended together according to Eq. 19 and are incorporated into the multi-objective model after normalizing through the direct method. γ and β represent the degree of importance of these two factors in relation to each other.

$$w_i = \gamma_i \times C_i + \beta_i \times D_i \qquad W_i = \frac{w_i}{\sum_{i=1}^{N} w_i} \tag{19}$$

Goal programming

GP is a multi-objective decision-making method which can conduct programming based upon the goals existing in each system and obtain satisfactory solutions for different goals. It is also possible to create deviations in the goals. GP incorporates goals, deviations, and goal and system limitations.

In this paper, in order to solve the multi-objective model, due to the allocation of a predetermined budget for the costs of reliability improvement in the design and production phases, and in order to prevent the integration of design and production costs, GP method is used. In fact, these budget restrictions are considered as deviation from goals. The model presented within the framework of GP is as follows:

$$\min Z = d_1^- / R_{\text{goal}} + d_2^+ / u_1 + d_3^+ / u_2 \tag{20}$$

$$s.t.$$

$$\prod_{i=1}^{N} R_i + d_1^- - d_1^+ = R_{\text{goal}} \tag{21}$$

$$\sum_{i=1}^{N} B1_i \cdot \left(e^{(1-Fd_i)\left(\frac{R_i - R_{\min}^{W_i}}{R_{\max}^{W_i} - R_i}\right)} - 1 \right) + d_2^- - d_2^+ = u_1 \tag{22}$$

$$\sum_{i=1}^{N} A_i + B2_i \cdot \left(R_i^{F_{p_i}} - R_{\min}^{w_i \times F_{p_i}} \right)(P/A, i, T) + d_3^- - d_3^+ = u_2 \tag{23}$$

$$\prod_{i=1}^{N} R_i \geq R_{\text{goal}} \qquad R_{\min}^{W_i} \leq R_i \leq R_{\max}^{W_i} \qquad 0 \leq R_i \leq 1 \tag{24}$$

Equation 20 demonstrates the minimization of deviation from goals. This equation is rendered dimensionless by dividing it by the amount of deviation. Equation 21 relates to the maximization of reliability, where d_1^- stands for negative deviation tendency. Equation 22 relates to the minimization of design cost, where d_2^+ represents positive deviation minimization. Equation 23 relates to the minimization of production costs, in which d_3^+ stands for the minimization of positive deviation. Equation 24 incorporates system limitations.

Practical example

In this paper, reliability allocation of electro-optical systems is investigated using the proposed model. As demonstrated in Fig. 4, the electro-optical system comprises six subsystems ($N = 6$). The system in question is a series system. The reliability of the system is computed through Eq. 25.

$$R_{\text{system}} = \prod_{i=1}^{6} R_i = R_1 \cdot R_2 \cdot R_3 \cdot R_4 \cdot R_5 \cdot R_6 \tag{25}$$

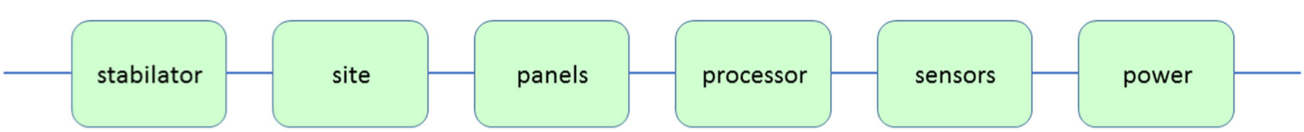

Fig. 4 Electro-optical system

Table 3 Values of each factor in each subsystem

	Complexity	Technology	Operational time	Environment and work condition	Safety	Repairability	Availability
Stabilator	9	7	6	8.5	3	8.5	8
Site	7	5	5	6.5	8	6	5.5
Panels	7	5	6	2.5	4.5	7	7
Processor	8	8	8	5.5	7	8	9
Sensors	8	6.5	8.5	10	2.5	5	7
Power	6	5	9	8	8	9	10

A questionnaire is used to select the effective factors among existing factors. Experts who know the system are asked to give a value between 0 and 100 for each factor. After calculating the frequency of each factor, Pareto chart is used for selecting effective factors. The effective feasibility factors in electro-optical systems include complexity, technology, operational time, environment and work condition, safety, repairability, and availability.

According to the relationship of each factor with reliability and experts opinion, the value between 1 and 10 is assigned for each factor in each subsystem. In order to simplify experts opinion, the mode of values φ_i is computed. In Table 3, simplified values of each factor are given in each subsystem. The weight of each selected factor is determined using MEMV-OWA solution in the Lingo software assuming $\alpha = 0.6$ and $n = 7$. Afterward, using Eq. 11, the weights of reliability improvement feasibility factor in the design phase for each subsystem are computed.

The data pertaining to the number of defects in the subsystems as well as existing failure opportunities were investigated for 10 units. Table 4 presents the values of these weights.

As presented in Table 4, the variation of reliability improvement feasibility factors in the design phase is consistent with those of the difficulty factor in the production phase. That is, systems with lower F_d (where reliability improvement feasibility is lower, reliability is higher, and the improvement costs approach infinity sooner) exhibited a more efficient performance in the production phase, except for the stabilator subsystem.

Table 5 presents the data pertaining to failure severity and the amount of efforts made to improve the subsystems. The C_i value is computed for each subsystem through Eq. 17.

Table 4 Result of calculating difficulty factor in each two phase

Subsystem	F_p	F_d
Stabilator	0.5	0.44
Site	0.41	0.36
Panels	0.48	0.33
Processor	0.25	0.45
Sensors	0.35	0.43
Power	0.39	0.41

The spatial and structural relationships, energy, and information among subsystems are based upon the structure of electro-optical systems. Using the paired comparison matrix, the weight of the importance of these relationships is computed. Figure 5 demonstrates the electro-optical system DSM.

As illustrated, there may be more than one relationship for each subsystem in each DSM cell. The relationships existing in the DSM are simplified, as in Fig. 6.

In what follows, the dependency weight for each subsystem is computed through Eq. 18. The two factors D_i and C_i are integrated in accordance with Eq. 19 and are incorporated into the model limitations after descaling in the direct fashion. The degree of importance of both items is considered as 0.5. Table 6 presents the final weight as well as design and production phase costs.

If $R_{\min} = 0.7$, $R_{\max} = 0.99$, $R_{\text{goal}} = 0.93$, $u_1 = 3000$, $u_2 = 600$, $T = 5$, and $i = 10\%$ per year, using the collected data, the GP model of reliability allocation is solved through Lingo 9 software program. The priorities considered for different objectives are identical. The results presented in the table are the global optimum solutions of the Lingo program. Table 7 presents the results of reliability allocation of the subsystems.

As demonstrated in Table 7 and Fig. 7, the reliability of subsystems has undergone an increase or a decrease according to production costs for improving reliability or the sigma level index. It is showed that sigma level index effect on results improves reliability of subsystems. In the main model, design costs and production costs equal 2945 and 513, respectively.

Table 5 Result of calculating criticality factor

Subsystem	s_i	e_i	C_i
Stabilator	0.05	0.20	0.193
Site	0.24	0.20	0.164
Panels	0.53	0.13	0.080
Processor	0.02	0.20	0.197
Sensors	0.05	0.13	0.189
Power	0.11	0.14	0.177

	stabilator		site		panels		processor		sensors		power	
stabilator			2	2								
								1		2	2	
site	1	1										
panels			2	2								
								2		2	2	
processor	1		1	1								
										2	2	
sensors			2	2								
								1			2	
power				1								

Fig. 5 DSM of electro-optical system

	stabilator	site	panels	processor	sensors	power	d_{ir}
stabilator		0.316	0.000	0.564	1.129	0.555	2.564
site	0.158		0.000	0.000	0.000	0.000	0.158
panels	0.000	0.316		1.129	1.129	0.555	3.129
processor	0.036	0.158	0.000		1.129	0.555	1.878
sensors	0.000	0.316	0.000	0.564		0.555	1.436
power	0.000	0.122	0.000	0.000	0.000		0.122
d_{is}	0.194	1.228	0.000	2.258	3.387	2.221	

Fig. 6 Simplifying the existing relationships in the DSM

Table 6 Final weights and costs of design and production phase

Subsystem	Di	W_i	A (million toman)	$B1$ (million toman)	$B2$ (million toman)
Stabilator	0.149	0.171	10	30	20
Site	0.075	0.119	15	20	5
Panels	0.168	0.124	20	20	10
Processor	0.223	0.210	50	100	150
Sensors	0.260	0.224	10	20	20
Power	0.126	0.151	20	50	80

Table 7 Comparison of subsystems reliability

Subsystem	Reliability of subsystems	
	Considering design cost	Considering design and production cost
Stabilator	0.98603	0.98552
Site	0.99197	0.99153
Panels	0.99096	0.99018
Processor	0.98338	0.98416
Sensors	0.98796	0.98759
Power	0.98759	0.98890
System	0.93000	0.93011

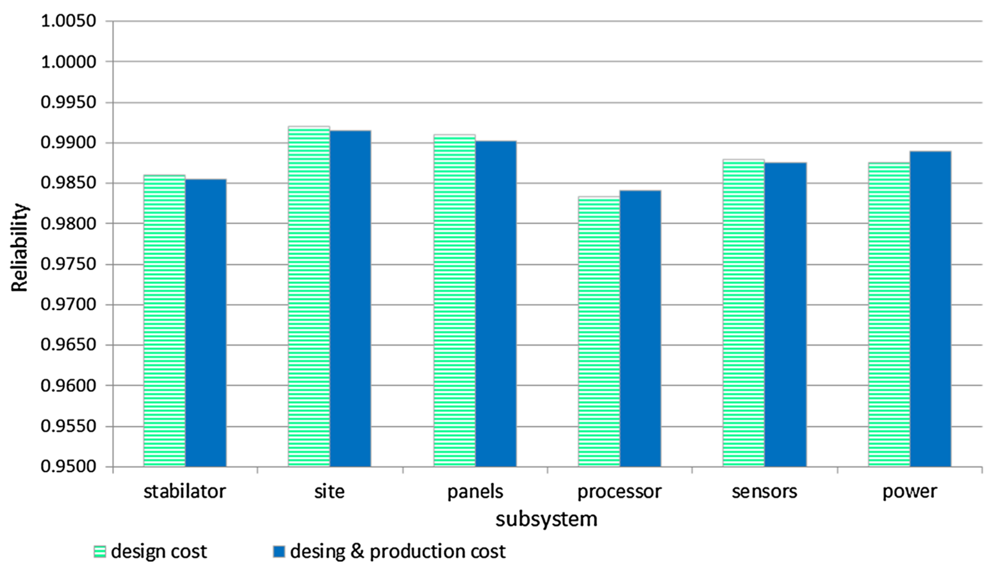

Fig. 7 Comparison of subsystems reliability by considering design and production cost and without considering production cost

Comparison with previous methods

In this section, the results of the proposed methods are compared with those of the previous methods. Table 8 presents the results of reliability of the subsystems of the electro-optical system in different methods.

The two methods FOO and MEMV-OWA allocate higher reliability to the subsystems with lower weights. Lower weights are determined for the subsystems with higher reliability. Therefore, these two methods fail to take into consideration difficulty and subsystem reliability improvement priorities, which makes them unable to assist managers and designer in rational decision-making.

In the modified criticality method and in the case of taking dependency into consideration, the factors of costs and reliability improvement feasibility are not investigated according to the properties and conditions of subsystems. The difference between the reliability values of these two methods and those of the proposed method lies in the incorporation of costs, feasibility factors, and system performance in the production phase in order to improve reliability.

Table 8 Comparison of results of proposed method with previous methods

Subsystem	Reliability of subsystems					
	FOO method	Chang et al. (2009)	Yadav and Zhuang (2014)	Zhuang et al. (2014)	Farsi and Jahromi (2012)	Proposed method
Stabilator	0.98379	0.98696	0.98611	0.98769	0.99000	0.98552
Site	0.99423	0.98927	0.98816	0.99138	0.99000	0.99153
Panels	0.99733	0.99011	0.99419	0.99101	0.99000	0.99018
Processor	0.98578	0.98659	0.98582	0.98490	0.99000	0.98416
Sensors	0.97777	0.98719	0.98636	0.98384	0.99000	0.98759
Power	0.98908	0.98776	0.98726	0.98907	0.99000	0.98890
System	0.93000	0.93000	0.93000	0.93000	0.94148	0.93011

The model developed by Farsi and Jahromi (2012) offers reliability close to maximum reliability, which leads to an exponential increase in costs.

Sensitivity analysis

R_{goal} represents important parameters in reliability problems. Table 9 and Fig. 8 present reliability values of subsystems for different R_{goal} values. As R_{goal} increases, subsystem reliability increases or remains constant and design and production costs increase.

Table 10 presents subsystem reliability values for the two cases of identicalness and non-identicalness of reliability improvement difficulty factors in the design and production phases.

As illustrated in Fig. 9, subsystems reliability has increased in all cases except in the stabilator case. In this subsystem, $F_d = 0.44$ and $F_p = 0.5$ and these values are higher than in other subsystems. Despite high F_d value, this

Table 9 Value of subsystems reliability with different goal reliability

Number	Subsystem	$R_{goal} = 0.87$	$R_{goal} = 0.91$	$R_{goal} = 0.97$
1	Stabilator	0.98044	0.98368	0.99499
2	Site	0.98396	0.98976	0.99622
3	Panels	0.98745	0.98890	0.99596
4	Processor	0.96731	0.97773	0.99357
5	Sensors	0.97482	0.98096	0.99364
6	Power	0.96845	0.98544	0.99525
	Design cost (million toman)	398	1087	1843355
	Production cost (million toman)	505	510	515

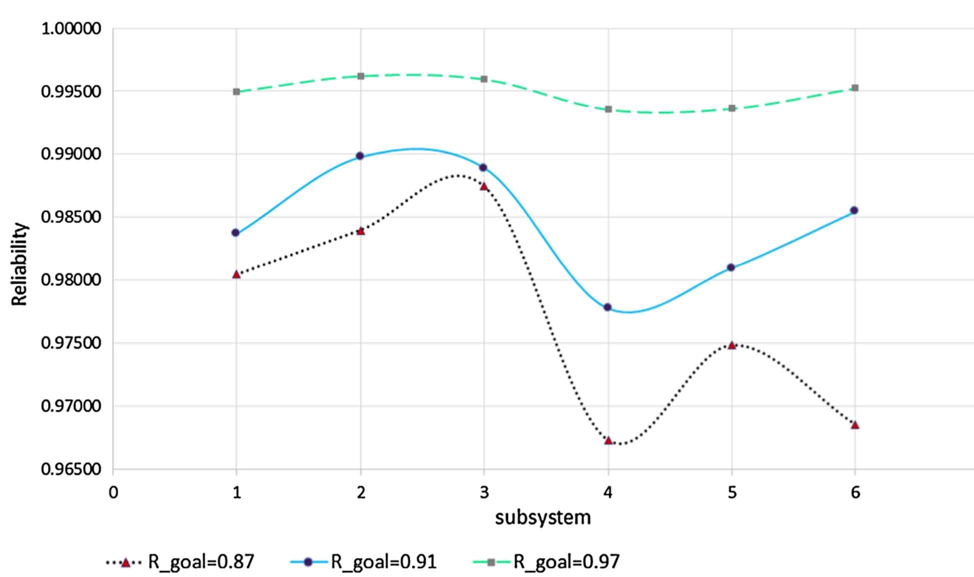

Fig. 8 Comparison of subsystems reliability with different goal reliability

Table 10 Effect of reliability improvement difficulty factor on the subsystems reliability

Subsystem	Real difficulty factor	Same difficulty factor	Level of reliability improvement
Stabilator	0.98552	0.99828	− 0.01276
Site	0.99153	0.99116	0.00037
Panels	0.99018	0.98992	0.00027
Processor	0.98416	0.98074	0.00342
Sensors	0.98759	0.98121	0.00638
Power	0.98890	0.98666	0.00224
System	0.93011	0.93000	

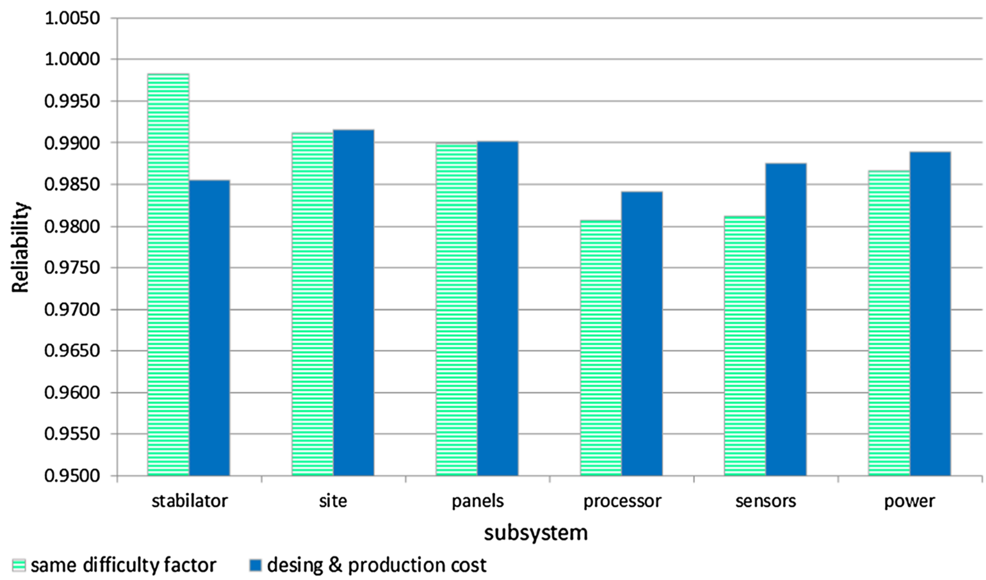

Fig. 9 Subsystems reliability by considering reliability improvement difficulty factor versus same difficulty factor

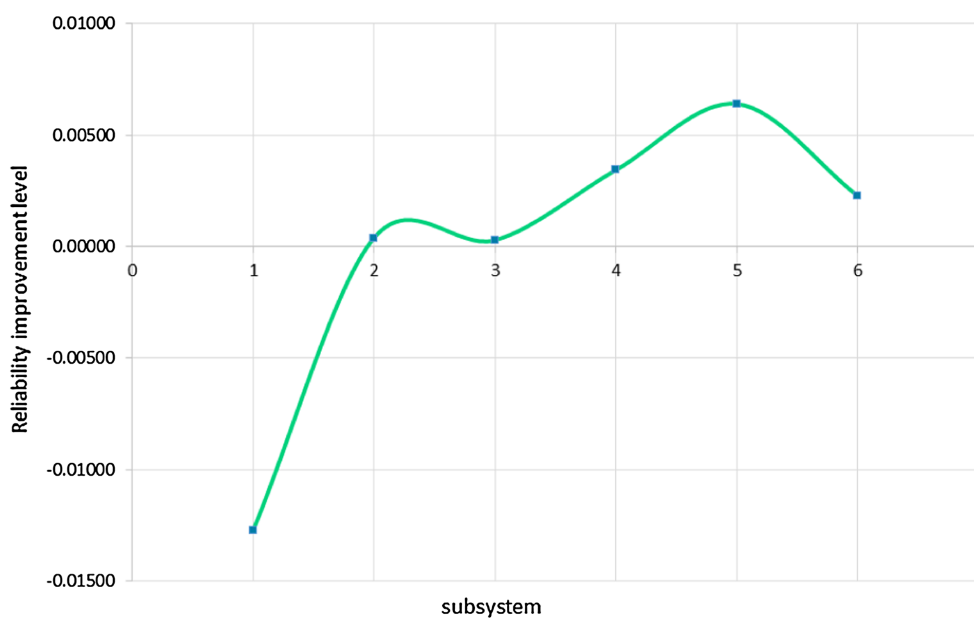

Fig. 10 Level of subsystems reliability improvement by considering difficulty factor

subsystem has enjoyed low reliability improvement due to high F_p value and cost reduction attempts. For the subsystem of site and panels, reliability improvement difficulty is higher in both phases, hence lower reliability improvement in comparison with other subsystems. This is illustrated in Fig. 10 which draws a comparison between the degrees of improvement. Considering F_p and F_d values for the processor, sensors, and power, greater improvement must have been considered, as illustrated in Fig. 10. Due to high costs of production, the degree of improvement in the processor is lower than that in the two other subsystems. Besides, as illustrated in Fig. 10, considering the degree of difficulty in the two phases, the degree of improvement in subsystems 4, 5, and 6 is higher than that of subsystems 1, 2, and 3. Hence, considering the results presented, reliability allocation is investigated by considering reliability improvement priorities, costs of design and production phases, and weight allocation restrictions.

Conclusion

The reliability of the final product is determined in the design phase and achieved in the production phase. The production phase is crucial for system reliability and the uncertainties existing in this stage. In a realistic reliability allocation method, the potential for reliability allocation must be investigated. Taking subsystem dependency into consideration in reliability allocation modifies allocation weights and links failure analysis process and reliability improvement difficulty to the product design and development process. In this paper, in order to allocate the reliability of subsystems, a multi-objective model was developed by maximizing whole system reliability and minimizing the costs of design and production phases. Four factors including reliability improvement in the design phase and production phase, criticality, and subsystem dependency were investigated. After converting the multi-objective model to GP and solving the model for the electro-optical system, the results were evaluated and analyzed. These results are consistent with the priorities of reliability improvement for each subsystem, and the difficulty factor used in the objective function investigates reliability improvement difficulty. These factors and costs tend to improve reliability considering the characteristics and function of subsystems, and improvement is subjected to criticality and dependency restrictions. In fact, in this method, reliability allocation is implemented in two stages: First, considering the allocation weight, a range is determined for the reliability of subsystems, and, afterward, improvement is implemented by considering the priorities of subsystem reliability improvement and the related costs. The proposed model provides greater flexibility for the engineers who seek to identify improvement opportunities and regulate reliability objectives with respect to reliability improvement feasibility, degree of criticality, and subsystem dependency. It is recommended that future studies investigate the effect of learning on production costs, study failure dependency among subsystems, investigate the proposed model for parallel and series–parallel systems, and examine other indices as factors of reliability improvement in the production phase and compare the result with this paper.

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