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ORIGINAL RESEARCH

Multi-choice stochastic bi-level programming problem in cooperative nature via fuzzy programming approach

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Abstract In this paper, a Multi-Choice Stochastic Bi-Level Programming Problem (MCSBLPP) is considered where all the parameters of constraints are followed by normal distribution. The cost coefficients of the objective functions are multi-choice types. At first, all the probabilistic constraints are transformed into deterministic constraints using stochastic programming approach. Further, a general transformation technique with the help of binary variables is used to transform the multi-choice type cost coefficients of the objective functions of Decision Makers(DMs). Then the transformed problem is considered as a deterministic multi-choice bi-level programming problem. Finally, a numerical example is presented to illustrate the usefulness of the paper.

Keywords Bi-level programming · Stochastic programming · Multi-choice programming · Fuzzy programming · Non-linear programming

Introduction and literature review

Bi-level programming problem under cooperative environment

Real-life decision-making problems in which there are multiple Decision Makers(DMs), who make decisions successively, are often formulated as a multi-level programming problems. Assuming that each DM makes a decision without any communication with some other DMs, as a solution concept to the problems, Stackelberg solution is employed. However, for decision-making problems in decentralized firms, it is quite natural to assume that there exist communication and some cooperative relationship among the DMs.

Anandalingam (1988) considered a mathematical programming model of decentralized multi-level systems and discussed the solution procedure. Anandalingam and Apprey (1991) proposed and discussed the multi-level programming with conflict resolution. Lai (1996) discussed hierarchical optimization and obtained a satisfactory solution for this multi-level programming. Sinha and Sinha (2004) considered linear multi-level programming under fuzzy environment. Dempe and Starostina (2007) considered a fuzzy bi-level programming problem and described the solution procedure with the help of multi-criteria optimization technique. In 2001, Roy (2001) proposed an approach to multi-objective bi-matrix games for Nash equilibrium solution. In 2006, Roy (2006) presented a fuzzy programming techniques for Stackelberg game. He used in his paper a fuzzy programming technique to solve Stackelberg game and compared the solution with the Kuhn–Tucker transformation technique. In 2007, Roy (2007) solved two-person multi-criteria bi-matrix games using fuzzy programming technique. Dey et al. (2014) presented a technique for order preference by similarity to ideal solution (TOPSIS) algorithm to linear fractional bi-level multi-objective decision-making problem in 2014. In 2012, Lachhwani and Poonia (2012) suggested for solving multi-level fractional programming problems in a large hierarchical decentralized organization using fuzzy goal programming approach. Zheng et al. (2011) discussed a class of bi-level multi-objective programming problem under fuzzy environment.

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Shih et al. (1996) proposed the multi-level programming problem with fuzzy approach and also discussed the solution concepts assuming cooperative communication among the DMs. Their methods were based on the idea that the DM at the lower level optimizes his or her objective function, taking a goal or preference of the DM at the upper level into consideration. The DM identifies the membership functions of fuzzy goals for their objective functions, and especially, the DM at the upper level also specifies those of fuzzy goals for decision variables. The DM at the lower level solves a fuzzy programming problem with constraints on a satisfactory degree of the DM at the upper level.

In this paper, we consider the multi-choice stochastic bi-level programming problem in cooperative environment and also assume that the DMs at the upper level and at the lower level have own fuzzy goals with respect to their objective functions.

The mathematical model of bi-level programming problem is as follows:

Model 1

$$\max_{\text{for DM}_{11}} Z_{11}(x) = \sum_{j=1}^{n} c_{11j}x_j$$

$$\max_{\text{for DM}_{2f}} Z_{2f}(x) = \sum_{j=1}^{n} c_{2fj}x_j$$

$$\text{subject to} \sum_{j=1}^{n} a_{ij}x_j \le b_i \quad i = 1, 2, \ldots, m,$$

$$x_j \ge 0 \quad j, \quad f = 1, 2, \ldots, n.$$

Stochastic programming

In most of the real-life decision-making problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extremum problems under incomplete information about the random parameters is called stochastic programming. Many problems in applied mathematics may be considered as belonging to any one of the following classes:

1. Descriptive problems, in which, with the help of mathematical methods, information is processed about the investigated event, some laws of the event being induced by others.
2. Optimization problems in which from a set of feasible solutions, an optimal solution is chosen.

Besides the above division of applied mathematical problems, these problems may be further classified as deterministic and stochastic problems. In the process of solution of the stochastic problem, several mathematical models have been developed. However, probabilistic methods were for a long time applied exclusively to the solution of the descriptive type of problems. Research on the theoretical development of stochastic programming has been going on for last four decades and to solve the several real-life problems in management science, it has been applied successfully. The chance constrained programming was first developed by Charnes and Cooper (1978).

Multi-choice programming

Multi-choice programming is a mathematical programming problem, in which DM is allowed to set multiple number of choices for a parameter. The situation of multiple choices for a parameter exists in many managerial decision-making problems. The multi-choice programming cannot only avoid the wastage of resources but also decide on the appropriate resource from multiple resources. A method for modeling the multi-choice programming problem, using binary variables was presented by Chang (2007). He has also proposed a revised method for multi-choice goal programming model which does not involve multiplicative terms of binary variables to model the multiple aspiration levels Chang (2008). Acharya and Acharya (2013) presented the generalized transformation technique for a multi-choice linear programming problems in which constraints are associated with some multi-choice parameters. Recently, Mahapatra et al. (2013) and Roy (2006) discussed the multi-choice stochastic transportation problem involving extreme value distribution and exponential distribution in which the multi-choice concept involved only in the cost parameters. In 2014, Maity and Roy (2014) presented also multi-choice multi-objective transportation problem using utility function approach. Recently, Maity and Roy (2015) studied a mathematical model for a transportation problem consisting of a multi-objective environment with non-linear cost and multi-choice demand. Roy (2015) discussed the solution procedure for multi-choice transportation problem using Langrange's interpolating polynomial approach. Roy (2014) presented multi-choice stochastic transportation problem involving Weibull distribution.

In this paper, we consider a generalized transformation technique to transform a multi-choice stochastic bi-level programming problem to an equivalent mathematical programming model. Using the transformation technique, the transformed model can be derived. Applying fuzzy programming technique, optimal solution of the proposed model is obtained.

The organization of the paper is as follows: following the introduction and literature review in Sect. 1, mathematical model of multi-choice stochastic bi-level programming problem (MCSBLPP) is presented in Sect. 2. Mathematical formulation is presented in Sect. 3 and solution procedure in Sect. 4. To verify the proposed methodology of the paper, a numerical example is presented in Sect. 5. In Sect. 6, the results of the given problems have been discussed here. Section 7 presents sensitivity analysis with our proposed problem. Finally, conclusion is presented in Sect. 8.

Mathematical model of MCSBLPP

In the mathematical model of Sect. 1, considering the cost coefficients of the objective functions for both DMs are multi-choice types and also assume that all the parameters of the constraints are random variables. Then the corresponding mathematical model of bi-level programming problem is to be treated as multi-choice stochastic bi-level programming problem (**MCSBLPP**) and is stated as below:

Model 2

$$\begin{aligned}
&\max_{\text{for DM}_{11}} Z_{11}(x) = \sum_{j=1}^{n}\left(c_{11j}^{(1)}, c_{11j}^{(2)}, \ldots, c_{11j}^{(k_j)}\right)x_j,\\
&\max_{\text{for DM}_{2f}} Z_{2f}(x) = \sum_{j=1}^{n}\left(c_{2fj}^{(1)}, c_{2fj}^{(2)}, \ldots, c_{2fj}^{(k_j)}\right)x_j,\\
&\text{subject to } P_r\left[\sum_{j=1}^{n} a_{ij}x_j \le b_i\right] \ge p_i \quad i = 1,2,\ldots,m,\\
&\text{where } x_j \ge 0 \quad f = 1,2,\ldots,n \quad 0 \le p_i \le 1; \quad \forall i,j.\\
&\text{and } p_i \text{ is the pre-specified level of probability.}
\end{aligned} \tag{1}$$

Model formulation

Assuming that $a_{ij}\ (i = 1,2,\ldots,m; j = 1,2,\ldots,n)$ and $b_i\ (i = 1,2,\ldots,m)$ are normal random variables, $c_{11j} = (c_{11j}^{(1)}, c_{11j}^{(2)}, \ldots, c_{11j}^{(k_j)})\ \forall j$ and $c_{2fj} = (c_{2fj}^{(1)}, c_{2fj}^{(2)}, \ldots, c_{2fj}^{(k_j)})\ \forall j$ are multi-choice parameters.

The following cases are to be considered:

1. Only $a_{ij}\ (i = 1,2,\ldots,m;\ j = 1,2,\ldots,n)$ follows normal distribution.
2. Only $b_i\ (i = 1,2,\ldots,m)$ follows normal distribution.
3. Both $a_{ij}\ (i = 1,2,\ldots,m;\ j = 1,2,\ldots,n)$ and $b_i\ (i = 1,2,\ldots,m)$ follow normal distributions.

Conversion of probabilistic constraints to deterministic constraints

Only $a_{ij}\ (i = 1,2,\ldots,m;\ j = 1,2,\ldots,n)$ follows Normal distribution.

Assuming that $\overline{a}_{ij}$ and $Var(a_{ij}) = \sigma_{a_{ij}}^2$ are the mean and variance of a_{ij}. Considering that the multivariate distribution of a_{ij} is also known along with the covariance, $cov(a_{ij}, a_{kl})$ between the random variables a_{ij} and a_{kl}. We consider f_i as

$$f_i = \sum_{j=1}^{n} a_{ij}x_j \quad i = 1,2,\ldots,m.$$

As x_j s are constants (not yet known), let the mean $\overline{f}_i$ and variance $Var(f_i)$ are defined as follows: $\overline{f}_i = \sum_{j=1}^{n}\overline{a}_{ij}x_j$ $(i = 1,2,\ldots,m)$ and $Var(f_i) = X^T V_i X$ where V_i is ith covariance matrix which is defined as follows:

$$V_i = \begin{pmatrix} Var(a_{i1}) & cov(a_{i1}, a_{i2}) & \ldots & cov(a_{i1}, a_{in}) \\ cov(a_{i2}, a_{i1}) & Var(a_{i2}) & \ldots & cov(a_{i2}, a_{in}) \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ cov(a_{in}, a_{i1}) & cov(a_{in}, a_{i2}) & \ldots & Var(a_{in}) \end{pmatrix}$$

The constraints of Eq. 1 can be rewritten as:

$$P_r[f_i \le b_i] \ge p_i \quad i = 1,2,\ldots,m,$$

$$\text{i.e.,}\ P_r\left[\frac{f_i - \overline{f}_i}{\sqrt{Var(f_i)}} \le \frac{b_i - \overline{f}_i}{\sqrt{Var(f_i)}}\right] \ge p_i \quad i = 1,2,\ldots,m$$

$$\text{Therefore, } P_r[f_i \le b_i] = \phi\left[\frac{b_i - \overline{f}_i}{\sqrt{Var(f_i)}}\right]. \tag{2}$$

where $\phi(x)$ represents the cumulative distribution function of the standard normal distribution evaluated at x. Defining e_i as $\phi(e_i) = p_i$.

Then the constraints in Eq. 2 can be stated as

$$\phi\left[\frac{b_i - \overline{f}_i}{\sqrt{Var(f_i)}}\right] \ge \phi(e_i) \quad i = 1,2,\ldots,m.$$

These inequalities will be satisfied only if

$$\frac{b_i - \overline{f}_i}{\sqrt{Var(f_i)}} \ge e_i \quad i = 1,2,\ldots,m,$$

$$\text{i.e.,}\ \overline{f}_i + e_i\sqrt{Var(f_i)} - b_i \le 0 \quad i = 1,2,\ldots,m.$$

Thus, finally, the probabilistic constraints (1) can be transformed into deterministic constraints as:

$$\sum_{j=1}^{n}\overline{a}_{ij}x_j + e_i\sqrt{X^T V_i X} - b_i \le 0 \quad i = 1,2,\ldots,m.$$

Thus, we obtain a multi-choice deterministic model (**Model 3**) as follows:

Model 3

$$\begin{aligned}
&\max_{\text{for DM}_{11}} Z_{11}(x) = \sum_{j=1}^{n}\left(c_{11j}^{(1)}, c_{11j}^{(2)}, \ldots, c_{11j}^{(k_j)}\right)x_j,\\
&\max_{\text{for DM}_{2f}} Z_{2f}(x) = \sum_{j=1}^{n}\left(c_{2fj}^{(1)}, c_{2fj}^{(2)}, \ldots, c_{2fj}^{(k_j)}\right)x_j,\\
&\text{subject to} \sum_{j=1}^{n}\overline{a}_{ij}x_j + e_i\sqrt{X^T V_i X} - b_i \le 0 \quad i = 1,2,\ldots,m,\\
&\text{where } x_j \ge 0 \quad \forall j,f.
\end{aligned} \tag{3}$$

Only b_i $(i = 1, 2, \ldots, m)$ follows normal distribution

Considering that $\overline{b}_i$ and $Var(b_i)$ are the mean and variance of b_i, the constraints of Eq. 1 can be rewritten as

$$P_r\left[\sum_{j=1}^{n} a_{ij}x_j \le b_i\right] = P_r\left[\frac{\sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i}{\sqrt{Var(b_i)}} \le \frac{b_i - \overline{b}_i}{\sqrt{Var(b_i)}}\right] = P_r\left[\frac{b_i - \overline{b}_i}{\sqrt{Var(b_i)}} \ge \frac{\sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i}{\sqrt{Var(b_i)}}\right] \ge p_i$$

$$i.e., 1 - P_r\left[\frac{b_i - \overline{b}_i}{\sqrt{Var(b_i)}} \le \frac{\sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i}{\sqrt{Var(b_i)}}\right] \ge p_i, \quad i.e., P_r\left[\frac{b_i - \overline{b}_i}{\sqrt{Var(b_i)}} \le \frac{\sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i}{\sqrt{Var(b_i)}}\right] \le 1 - p_i. \quad (4)$$

Defining e_i as $\phi(e_i) = 1 - p_i$, the constraints in Eq. 4 can be stated as

$$\phi\left[\frac{\sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i}{\sqrt{Var(b_i)}}\right] \le \phi(e_i) \quad i = 1, 2, \ldots, m.$$

This inequality will be satisfied only if

$$\frac{\sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i}{\sqrt{Var(b_i)}} \le e_i \quad i = 1, 2, \ldots, m.$$

Thus, finally, the probabilistic constraints (1) can be transformed into deterministic constraints as:

$$\sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i - e_i\sqrt{Var(b_i)} \le 0 \quad i = 1, 2, \ldots, m.$$

Thus, we have obtained a multi-choice deterministic model (**Model 4**) as follows:

Model 4

$$\max_{\text{for DM}_{11}} Z_{11}(x) = \sum_{j=1}^{n}\left(c_{11j}^{(1)}, c_{11j}^{(2)}, \ldots, c_{11j}^{(k_j)}\right)x_j,$$

$$\max_{\text{for DM}_{2f}} Z_{2f}(x) = \sum_{j=1}^{n}\left(c_{2fj}^{(1)}, c_{2fj}^{(2)}, \ldots, c_{2fj}^{(k_j)}\right)x_j,$$

$$\text{subject to} \sum_{j=1}^{n} a_{ij}x_j - \overline{b}_i - e_i\sqrt{Var(b_i)} \le 0 \quad i = 1, 2, \ldots, m,$$

$$\text{where } x_j \ge 0 \quad \forall j, f. \quad (5)$$

Both a_{ij} $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ and b_i $(i = 1, 2, \ldots, m)$ follow normal distribution

Define a random variable h_i as

$$h_i = \sum_{j=1}^{n} a_{ij}x_j - b_i = \sum_{k=1}^{n+1} q_{ik}y_k,$$

where $q_{ik} = a_{ik}, \quad q_{i,n+1} = b_i$.
and $y_k = x_k, \quad y_{n+1} = -1; \quad k = 1, 2, \ldots, n+1$.

The constraints of Eq. 1 can be rewritten as:

$$P_r[h_i \le 0] \ge p_i \quad i = 1, 2, \ldots, m. \quad (6)$$

The mean $\overline{h}_i$ and variance of $Var(h_i)$ are given by

$$\overline{h}_i = \sum_{k=1}^{n+1} \overline{q}_{ik}y_k = \sum_{j=1}^{n} \overline{a}_{ij}x_j - \overline{b}_i,$$

and $Var(h_i) = X^T V_i X$ where V_i is given by

$$V_i = \begin{pmatrix} Var(a_{i1}) & cov(a_{i1}, a_{i2}) & \ldots & cov(a_{i1}, a_{in}) \\ cov(a_{i2}, a_{i1}) & Var(a_{i2}) & \ldots & cov(a_{i2}, a_{in}) \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ cov(a_{in}, a_{i1}) & cov(a_{in}, a_{i2}) & \ldots & Var(a_{in}) \end{pmatrix},$$

This can be also rewritten as:

$$\begin{aligned} Var(h_i) &= \sum_{k=1}^{n+1}\left[y_k^2 Var(q_{ik}) + 2\sum_{l=k+1}^{n+1} y_k y_l cov(q_{ik}, q_{il})\right] \\ &= \sum_{k=1}^{n}\left[y_k^2 Var(q_{ik}) + 2\sum_{l=k+1}^{n} y_k y[_l cov(q_{ik}, q_{il})\right] \\ &\quad + y_{n+1}^2 Var(q_{i,n+1}) + 2y_{n+1}^2 cov(q_{i,n+1}, q_{i,n+1} \\ &\quad + \sum_{k=1}^{n}\left[2y_k y_{n+1} cov(q_{ik}, q_{i,n+1})\right] \\ &= \sum_{k=1}^{n}\left[x_k^2 Var(a_{ik}) + 2\sum_{l=k+1}^{n} x_k x_l cov(a_{ik}, a_{il})\right] \\ &\quad + Var(b_i) - 2\sum_{k=1}^{n} x_k cov(a_{ik}, b_i). \end{aligned}$$

Thus, the constraints in Eq. 6 can be restated as follows:

$$P_r\left[\frac{h_i-\overline{h}_i}{\sqrt{Var(h_i)}}\le\frac{-\overline{h}_i}{\sqrt{Var(h_i)}}\right]\ge p_i\quad i=1,2,\ldots,m.$$
$$\text{Therefore}, P_r\left[h_i\le 0\right]=\phi\left[\frac{-\overline{h}_i}{\sqrt{Var(h_i)}}\right]. \tag{7}$$

Defining e_i as $\phi(e_i)=p_i$, and then the constraints in equation (7) can be rewritten as follows:

$$\phi\left[\frac{-\overline{h}_i}{\sqrt{Var(h_i)}}\right]\ge\phi(e_i)\quad i=1,2,\ldots m.$$

This inequality will be satisfied only if

$$\frac{-\overline{h}_i}{\sqrt{Var(h_i)}}\ge e_i\quad i=1,2,\ldots m,$$
$$\text{i.e.,}\ \overline{h}_i+e_i\sqrt{Var(h_i)}\le 0\quad i=1,2,\ldots,m.$$

Thus, finally, the probabilistic constraints (1) can be transformed into a deterministic constraints as $\sum_{j=1}^{n}\overline{a}_{ij}x_{ij}-\overline{b}_i+e_i\sqrt{X^TV_iX}\le 0\quad i=1,2,\ldots m$. Thus, we obtain a multi-choice deterministic model (**Model 5**) as follows:

Model 5

$$\max_{\text{for DM}_{11}} Z_{11}(x)=\sum_{j=1}^{n}\left(c_{11j}^{(1)},c_{11j}^{(2)},\ldots,c_{11j}^{(k_j)}\right)x_j,$$
$$\max_{\text{for DM}_{2f}} Z_{2f}(x)=\sum_{j=1}^{n}\left(c_{2fj}^{(1)},c_{2fj}^{(2)},\ldots,c_{2fj}^{(k_j)}\right)x_j,$$
$$\text{subject to}\sum_{j=1}^{n}\overline{a}_{ij}x_j-\overline{b}_i+e_i\sqrt{X^TV_iX}\le 0\quad i=1,2,\ldots,m.$$
$$\text{where}\ x_j\ge 0\quad\forall j,f. \tag{8}$$

Transformation of the objective functions involving multi-choice cost parameters

Now we present a transformation technique of MCSBLPP to formulate an equivalent mathematical model.

Step 1: Find the total number of choices from upper level and lower level decision maker's objective functions. Consider the total number of choices for upper level objective function is k_j. Suppose that $k_j\ge 2$.

Step 2: Find the number of binary variables, which is required to handle the multi-choice parameters in the following manner.

Find l_j, for which $2^{(l_j-1)}<k_j\le 2^{l_j}$. Here l_j number of binary variables are needed. Let the binary variables be $z_j^{(1)},z_j^{(2)},\ldots,z_j^{(l_j)}$.

Step 3: Express $2^{l_j}=\binom{l_j}{0}+\binom{l_j}{1}+\cdots+\binom{l_j}{r_{j_1}}+\ldots+\binom{l_j}{r_{j_2}}+\cdots+\binom{l_j}{l_j}$, and select the smallest number of consecutive terms whose sum is equal to or just greater than k_j from the expansion.

Let the terms be $\binom{l_j}{r_{j_1}},\binom{l_j}{r_{j_1+1}},\ldots,\binom{l_j}{r_{j_2}}$.

Step 4: Set k_j binary codes to k_j number of choices as follows:

$$\max_{\text{for DM}_{11}} Z_{11}(x) = \sum_{j=1}^{n} \left[\sum_{t=1}^{\binom{l_j}{r_{j_1}}} P_t^{(r_{j_1})} Q_t^{(r_{j_1})} c_{11j}^{(t)} + \sum_{t=1}^{\binom{l_j}{r_{j_1+1}}} P_t^{(r_{j_1+1})} Q_t^{(r_{j_1+1})} c_{11j}^{(\binom{l_j}{r_{j_1}}+t)} \right.$$

$$\left. + \cdots \sum_{t=1}^{\binom{l_j}{r_{j_2-1}}} P_t^{(r_{j_2-1})} Q_t^{(r_{j_2-1})} c_{11j}^{(\binom{l_j}{r_{j_1}}+\binom{l_j}{r_{j_1+1}}+\cdots+\binom{l_j}{r_{j_2-2}}+t)} + \sum_{t=1}^{(k_j-N_j^{(1)})} P_t^{(r_{j_2})} Q_t^{(r_{j_2})} c_{11j}^{(N_j^{(1)}+t)} \right] x_j$$

$$\max_{\text{for DM}_{2f}} Z_{2f}(x) = \sum_{j=1}^{n} \left[\sum_{t=1}^{\binom{l_j}{r_{j_1}}} P_t^{(r_{j_1})} Q_t^{(r_{j_1})} c_{2fj}^{(t)} + \sum_{t=1}^{\binom{l_j}{r_{j_1+1}}} P_t^{(r_{j_1+1})} Q_t^{(r_{j_1+1})} c_{2fj}^{(\binom{l_j}{r_{j_1}}+t)} \right.$$

$$\left. + \cdots \sum_{t=1}^{\binom{l_j}{r_{j_2-1}}} P_t^{(r_{j_2-1})} Q_t^{(r_{j_2-1})} c_{2fj}^{(\binom{l_j}{r_{j_1}}+\binom{l_j}{r_{j_1+1}}+\cdots+\binom{l_j}{r_{j_2-2}}+t)} + \sum_{t=1}^{(k_j-N_j^{(1)})} P_t^{(r_{j_2})} Q_t^{(r_{j_2})} c_{2fj}^{(N_j^{(1)}+t)} \right] x_j$$

where $N_j^{(1)} = \binom{l_j}{r_{j_1}} + \binom{l_j}{r_{j_1+1}} + \cdots + \binom{l_j}{r_{j_2-1}}$

$$t_1 \in \{1, 2, 3, \ldots, (l_j - s) + 1\};\; t_2 \in \{2, 3, \ldots, (l_j - s) + 2\}, \ldots,\; t_s \in \{s, s+1, \ldots, l_j\}$$

$$f = 1, 2, \ldots, n;$$

$$I_s(t) = \{\{t_1, t_2, \ldots, t_s\} \mid t_1 < t_2 < \cdots < t_s,\; s = r_{j_1}, r_{j_1} + 1, \ldots, r_{j_2}\}$$

$$P_t^{s_j} = \{z_j^{(t_1)} z_j^{(t_2)} z_j^{(t_3)} \ldots z_j^{(t_s)} \mid \{t_1, t_2, \ldots, t_s\} \in I_s(t),\; s = r_{j_1}, r_{j_1} + 1, \ldots, r_{j_2}\}$$

$$Q_t^{s_j} = \left\{ \prod_{t=1}^{l_j} (1 - z_j(t)) \mid t \notin \{t_1, t_2, \ldots, t_s\} \right\}$$

Step 5: Restrict $(2^{l_j} - k_j)$ number of binary codes to overcome repetitions as follows:

$$z_j(1) + z_j(2) + \ldots + z_j(l_i) \geq r_{j_1}$$
$$z_j(1) + z_j(2) + \cdots + z_j(l_i) \leq r_{j_2}$$
$$z_j(t_1) + z_j(t_2) + \cdots + z_j(t_{r_{j_2}}) \leq r_{j_2-1},$$
$$t = (k_j - N_j(1)) + 1, (k_j - N_j(1)) + 2, \cdots, \binom{l_j}{r_{j_2}}$$

Restrictions should be imposed on $z_j^{(t_1)} z_j^{(t_2)} z_j^{(t_3)} \ldots z_j^{(t_{r_{j2}})} \in P_t^{(r_{j_2} j)}$

Step 6: Formulate the mathematical model and this model is denoted by **Model 6** as follows:

Model 6

$$\begin{cases} \max\limits_{\text{for DM}_{11}} Z_{11}(x) = \sum\limits_{j=1}^{n}\Bigg[\sum\limits_{t=1}^{\binom{l_j}{r_{j_1}}} P_t^{(r_{j_1})} Q_t^{(r_{j_1})} c_{11j}^{(t)} + \sum\limits_{t=1}^{\binom{l_j}{r_{j_1+1}}} P_t^{(r_{j_1+1})} Q_t^{(r_{j_1+1})} c_{11j}^{(\binom{l_j}{r_{j_1}}+t)} + \cdots \\ \sum\limits_{t=1}^{\binom{l_j}{r_{j_2-1}}} P_t^{(r_{j_2-1})} Q_t^{(r_{j_2-1})} c_{11j}^{(\binom{l_j}{r_{j_1}}+\binom{l_j}{r_{j_1+1}}+\cdots+\binom{l_j}{r_{j_2-2}}+t)} + \sum\limits_{t=1}^{(k_j-N_j^{(1)})} P_t^{(r_{j_2})} Q_t^{(r_{j_2})} c_{11j}^{(N_j^{(1)}+t)} \Bigg] x_j \end{cases}$$

$$\begin{cases} \max\limits_{\text{for DM}_{2f}} Z_{2f}(x) = \sum\limits_{j=1}^{n}\Bigg[\sum\limits_{t=1}^{\binom{l_j}{r_{j_1}}} P_t^{(r_{j_1})} Q_t^{(r_{j_1})} c_{2fj}^{(t)} + \sum\limits_{t=1}^{\binom{l_j}{r_{j_1+1}}} P_t^{(r_{j_1+1})} Q_t^{(r_{j_1+1})} c_{2fj}^{(\binom{l_j}{r_{j_1}}+t)} + \cdots \\ \sum\limits_{t=1}^{\binom{l_j}{r_{j2-1}}} P_t^{(r_{j2-1})} Q_t^{(r_{j2-1})} c_{2fj}^{(\binom{l_j}{r_{j1}}+\binom{l_j}{r_{j1+1}}+\cdots+\binom{l_j}{r_{j2-2}}+t)} + \sum\limits_{t=1}^{(k_j-N_j^{(1)})} P_t^{(r_{j2})} Q_t^{(r_{j2})} c_{2fj}^{(N_j^{(1)}+t)} \Bigg] x_j \end{cases}$$

$$\begin{cases} z_j(1) + z_j(2) + \cdots + z_j(l_j) \ge r_{j_1} \\ z_j(1) + z_j(2) + \cdots + z_i(l_j) \le r_{j_2} \\ z_j(t_1) + z_j(t_2) + \cdots + z_j(t_{r_{j_2}}) \le r_{j_2-1}\ , \\ \quad t = (k_j - N_j(1)) + 1, (k_j - N_j(1)) + 2, \ldots, \binom{l_j}{r_{j_2}} \\ z_j^{(l_j)} = 0/1\ , l_j = 1, 2, \ldots, \lceil \frac{\ln k_j}{\ln 2} \rceil \quad j = 1, 2, \ldots, n \\ \text{where } N_j^{(1)} = \binom{l_j}{r_{j1}} + \binom{l_j}{r_{j1+1}} + \cdots + \binom{l_j}{r_{j2-1}} \end{cases} \tag{9}$$

subject to $\sum_{j=1}^{n} \overline{a}_{ij} x_j - \overline{b}_i + e_i\sqrt{X^T V_i X} \le 0 \quad i = 1, 2, \ldots, m;$ $x_j \ge 0, \forall j$

or $S = \{x_j, \forall j : \sum_{j=1}^{n} \overline{a}_{ij} x_j - \overline{b}_i + e_i\sqrt{X^T V_i X} \le 0 \quad i = 1, 2, \ldots, m; \quad x_j \ge 0, \forall j\}$

Step 7: Mathematical **Model 6** is a mixed integer non-linear programming problem. Solve the model with the help of LINGO 13.0 packages.

Solution procedure

Basic concepts of fuzzy set and membership function

Fuzzy set was first introduced by Zadeh in 1965 on a mathematical way to represent impreciseness or vagueness in everyday life.

Fuzzy set: A fuzzy set A in a discourse X is defined as the following set of pairs $A = \{(x, \mu_A) : x \in A\}$, where $\mu_A : X \to [0, 1]$ is a mapping, called membership function of fuzzy set A and μ_A is called the membership value or degree of membership of $x \in X$ in the fuzzy set A. The larger μ_A is the stronger grade of membership form in A.

Normal fuzzy set: Let A be a fuzzy set in X. The height $h(A)$ of A is defined as

$$h(A) = Sup\{\mu_A(x)\}.$$

If $h(A) = 1$, then fuzzy set is called a normal fuzzy set, otherwise it is called subnormal.

$\alpha -$ **cut**: Let A be a fuzzy set in X and $\alpha \in (0, 1]$. The $\alpha - cut$ of fuzzy set A in crisp set A_α given by

$$A_\alpha = \{x \in X : \mu_A(x) \ge \alpha\}.$$

Convex fuzzy set: A fuzzy set A in R^n is said to be a convex fuzzy set if its $\alpha - cut\, A_\alpha$ are (crisp) bounded sets, $\forall \alpha \in (0, 1]$.

Fuzzy number: Let A be a fuzzy set in **R** (set of real numbers). Then A is called a fuzzy number if

(i) A is normal,
(ii) A is convex,
(iii) μ_A is upper semicontinuous, and,
(iv) the support of A is bounded.

Triangular fuzzy number: A fuzzy number A is called a triangular fuzzy number(**TFN**) if its membership function μ_A is given by

$$\mu_A(x) = \begin{cases} 0, & \text{if } x < a_l, \quad x > a_u, \\ \dfrac{x - a_l}{a - a_l}, & \text{if } a_l \le x \le a, \\ \dfrac{a_u - x}{a_u - a}, & \text{if } a < x \le a_u. \end{cases} \quad (10)$$

The TFN A is denoted by the triplet $A = (a_l, a, a_u)$.

Fuzzy programming

In fuzzy programming, we construct the linear membership functions which are defined as:

$$\mu_{11}(Z_{11}(x)) = \begin{cases} 0, & \text{if } Z_{11}(x) > Z_{11}^0, \\ \dfrac{Z_{11}(x) - Z_{11}^1}{Z_{11}^0 - Z_{11}^1}, & \text{if } Z_{11}^1 < Z_{11}(x) \le Z_{11}^0, \\ 1, & \text{if } Z_{11}(x) \le Z_{11}^1, \end{cases} \quad (11)$$

$$\mu_{2f}(Z_{2f}(x)) = \begin{cases} 0, & \text{if } Z_{2f}(x) > Z_{2f}^0, \\ \dfrac{Z_{2f}(x) - Z_{2f}^1}{Z_{2f}^0 - Z_{2f}^1}, & \text{if } Z_{2f}^1 < Z_{2f}(x) \le Z_{2f}^0, \\ 1, & \text{if } Z_{2f}(x) \le Z_{2f}^1, \\ & \text{where } f = 1, 2, \ldots, n. \end{cases} \quad (12)$$

Zimmermann (1978) suggested a method for assessing the parameters of the membership function. In his method, the parameters Z_{11}^0, Z_{11}^1 and $Z_{2f}^0, Z_{2f}^1 \quad \forall f$ are determined as:

$Z_{11}^0 = \max_{x \in S} Z_{11}(x),\ Z_{11}^1 = \min_{x \in S} Z_{11}(x)$ and

$Z_{2f}^0 = \max_{x \in S} Z_{2f}(x) \quad \forall f = 1, 2, \ldots, n \quad , Z_{2f}^1 = \min_{x \in S} Z_{2f}(x)$

$\forall f = 1, 2, \ldots, n$, where S is the feasible region of **Model 6**.

Now, to give an algorithm of fuzzy programming technique for deriving a compromise solution to **Model 2**, which is summarized in the following way:

Step 1: Solve the objective function of the upper level decision maker (i.e., leader) and the lower level decision makers (i.e., followers) with constraints (8) independently.

Step 2: Calculate the linear membership functions in equations (10) and (11) for DM_{11} and DM_{2f}, $\forall f$.

Step 3: Solve the problem which is defined as follows:

Model 7

max λ

subject to

$$\begin{cases} \mu_{11}(Z_{11}(x)) \ge \lambda \\ \mu_{2f}(Z_{2f}(x)) \ge \lambda \\ \text{and Eq. } 8 \\ x \ge 0,\ f = 1, 2, \ldots, n \end{cases} \quad (13)$$

in which a smaller satisfactory degree between those of DM_{11} and DM_{2f} is maximized. If DM_{11} is satisfied with the obtained optimal solution, the solution becomes a satisfactory solution. Otherwise, DM_{11} is to specify the minimal satisfactory level δ together with the lower and upper bounds $[\Delta_{min}, \Delta_{max}]$ of the ratio of satisfactory degree Δ, where $\Delta = \max\{\frac{\mu_{2f}(Z_{2f}(x))}{\mu_{11}(Z_{11}(x))}, \forall f\}$ with the satisfactory degree $\lambda^* (= \min\{\mu_{11}(Z_{11}(x^*)), \mu_{2f}(Z_{2f}(x^*))\}$, $\forall f$ and x^* is an optimal solution of **Model 6**) of DMs and the related information about the solution in mind.

Step 4: Solve the problem which is defined as follows:

Model 8

max $\mu_{2f}(Z_{2f}(x))$

subject to

$$\begin{cases} \mu_{11}(Z_{11}(x)) \ge \delta \\ \text{and Eq. } 8 \\ x \ge 0,\ f = 1, 2, \ldots, n \end{cases} \quad (14)$$

in which the satisfactory degree of DM_{2f} is maximized under the condition that the satisfactory degree of DM_{11} is larger than or equal to the minimal satisfactory level δ, and then an optimal solution x in equation (12) is proposed to DM_{11} together with λ, $\mu_{11}(Z_{11})$, $\mu_{2f}(Z_{2f})$, $\forall f$ and Δ.

Step 5: If the solution x satisfies one of the following two conditions and DM_{11} accepts it, then goto **Step 7** and the solution x is determined to be the satisfactory solution.

5.1: DM_{11}'s satisfactory degree is larger than or equal to the minimal satisfactory level δ specified by DM_{11}'s self, i.e., $\mu_{11}(Z_{11}(x)) \ge \delta$.

5.2: The ratio Δ of satisfactory degrees lies in between the Δ_{min} and Δ_{max}, i.e., $\Delta \in [\Delta_{min}, \Delta_{max}]$.

Step 6: Ask DM_{11} to revise the minimal satisfactory level δ in accordance with the following procedure of updating the minimal satisfactory level.

6.1: If **Step 5.1** is not satisfied, then DM_{11} decreases the minimal satisfactory level δ.

6.2: If the ratio Δ exceeds its upper bound, then DM_{11} increases the minimal satisfactory level δ. Conversely, if Δ is below its lower bound, then DM_{11} decreases the minimal satisfactory level δ.

6.3: Although **Steps 5.1** and **5.2** are satisfied, if DM_{11} is not satisfied with the obtained solution and judges that it is desirable to increase the satisfactory degree of DM_{11} at the expense of the satisfactory degree of DM_{2f}, $\forall f$, then DM_{11} increases the minimal satisfactory level δ. Conversely, if DM_{11} judges that it is desirable to increase the satisfactory degree of DM_{2f}, $\forall f$ at the expense of the satisfactory degree of DM_{11}, then DM_{11} decreases the minimal satisfactory level δ.

Step 7: Stop.

Numerical example

A reputed industry farming organization operates six farms which are located at Bankura, Purulia, Bardwan, East Midnapur, West Midnapur and Nadia in West Bengal of India of comparable productivity. These farms planted two types of crops: rice and wheat, respectively. The output of each farm is limited both by the usable acreage and by the amount of water available for irrigation. Considering those parameters: usable acreage and water availability both follow a normal distribution. The data for the upcoming season is as shown below:

Farms	Usable Acreage		Minimum water available (in cubic feet)	
	Mean	Variance	Mean	Variance
Bankura	10	6	75	52
Purulia	13	7	78	55
Bardwan	15	8	95	62
Nadia	11	6.5	72	49
East Midnapur	12	6.6	80	59
West Midnapur	18	9	120	93

The organization is considering planting crops which differ primarily in their expected profits per acre and in their consumption of water and the profit to be considered in multi-choice type. Furthermore, the total acreage that can be devoted to each of the crops is limited by the amount of appropriate harvesting equipment available. The organization wishes to know how much of each crop should be planted at the respected farms to maximize the expected profit as well as the maximum revenue earned by the West Bengal Govt. Due to fluctuation of season in West Bengal, revenue is multi-choice type and also the expected revenue from these crops is (40, 42, 45) and (28, 30), respectively.

Crops	Maximum acreage		Water consumption (in cubic feet)		Expected profit (per acre)
	Mean	Variance	Mean	Variance	
Rice	70	52	65	48	(25000,26000,30000)
Wheat	50	41	50	41	(40000,45000)

Let $\mathbf{x_1}$, i.e., $x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}$ and $\mathbf{x_2}$, i.e., $x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}$ be the number of acres to be allocated for rice and wheat crops to the six firms located at Bankura, Purulia, Bardwan, East Midnapur, West Midnapur and Nadia, respectively. There are two different level decision makers with respect to this problem, i.e., government (i.e., leader) and the manager of industry (i.e., follower) and each one controls only one decision variable. The government controls rice crop (i.e., $\mathbf{x_1}$) in the first level and the manager controls wheat crop (i.e., $\mathbf{x_2}$) in the second level. Two objectives are established, respectively: (i) revenue from the profits $Z_{11}(\mathbf{x_1}, \mathbf{x_2})$ and (ii) profit on the cultivation of crops $Z_{21}(\mathbf{x_1}, \mathbf{x_2})$.

This is clearly a multi-choice stochastic bi-level programming problem. The problem cannot be solved without using multi-choice programming and stochastic programming approaches.

Using the methodology presented in Sect. 3.2, first we convert the multi-choice objective functions into deterministic objective function and again using Sect. 3.1, we convert the probabilistic constraints into deterministic constraints and then the whole problem is transformed as follows:

$$\max_{\text{forDM}_{11}} : Z_{11}(\mathbf{x_1}, \mathbf{x_2}) = \left[40z_1z_2 + 42z_1 \quad (1-z_2) + 45 (1-z_1)z_2\right] \quad (x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}) + \left[28z_3 + 30\ (1-z_3)\right](x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}),$$

$$\max_{\text{forDM}_{21}} : Z_{21}(\mathbf{x_1}, \mathbf{x_2}) = \left[25000z_4z_5 + 26000z_4(1-z_5) + 30000(1-z_4)z_5\right] \quad (x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}) + \left[40000z_6 + 45000(1-z_6)\right] \quad (x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}),$$

subject to

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16} - 2.33(52)^{\frac{1}{2}} \le 70,$$
$$x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26} - 2.33(41)^{\frac{1}{2}} \le 50,$$
$$x_{11}+x_{21} - 2.33(6)^{\frac{1}{2}} \le 10,$$
$$x_{12}+x_{22} - 2.33(7)^{\frac{1}{2}} \le 13,$$
$$x_{13}+x_{23} - 2.33(8)^{\frac{1}{2}} \le 15,$$
$$x_{14}+x_{24} - 2.33(6.5)^{\frac{1}{2}} \le 11,$$
$$x_{15}+x_{25} - 2.33(6.6)^{\frac{1}{2}} \le 12,$$
$$x_{16}+x_{26} - 2.33(9)^{\frac{1}{2}} \le 18,$$
$$65x_{11} + 50x_{21} + 2.33\left(48x_{11}^2 + 41x_{21}^2 + 52\right)^{\frac{1}{2}} \ge 75,$$
$$65x_{12} + 50x_{22} + 2.33\left(48x_{12}^2 + 41x_{22}^2 + 55\right)^{\frac{1}{2}} \ge 78,$$
$$65x_{13} + 50x_{23} + 2.33\left(48x_{13}^2 + 41x_{23}^2 + 62\right)^{\frac{1}{2}} \ge 95,$$
$$65x_{14} + 50x_{24} + 2.33\left(48x_{14}^2 + 41x_{24}^2 + 49\right)^{\frac{1}{2}} \ge 72,$$
$$65x_{15} + 50x_{25} + 2.33\left(48x_{15}^2 + 41x_{25}^2 + 59\right)^{\frac{1}{2}} \ge 80,$$
$$65x_{16} + 50x_{26} + 2.33\left(48x_{16}^2 + 41x_{26}^2 + 93\right)^{\frac{1}{2}} \ge 120,$$

$1 \le z_1 + z_2 \le 2,$
$1 \le z_4 + z_5 \le 2,$
where $x_{1j}, x_{2j} \ge 0 \quad j = 1, 2, \ldots, 6.$

Results and discussion

The above mathematical programming model is treated as non-linear programming problem and is solved by Lingo 13.0 packages. The results of the optimal solution to individual problems are obtained as:

$Z_{11}^1 = 204.82$ at $x_{1j} = 0\, j = 1, 2, \ldots, 6$, $x_{21} = 1.04$, $x_{22} = 1.09, x_{23} = 1.35, x_{24} = 1, x_{25} = 1.11, x_{26} = 1.72$ and the control variables are $z_1 = z_2 = z_3 = 1$; $Z_{11}^0 = 4532.97$ at $x_{11} = 13.14, x_{12} = 13.78, x_{13} = 7.94, x_{14} = 13.37, \quad x_{15} = 13.56, x_{16} = 24.99, x_{21} = 2.56, \quad x_{22} = 5.38, x_{23} = 3.57, x_{24} = 3.57, x_{25} = 4.42, x_{26} = 0$ and the control variables are $z_1 = 1, z_2 = z_3 = 0$;

$Z_{21}^1 = 15047.83$ at $x_{11} = 0.82, x_{12} = 0.86, x_{13} = 1.07, x_{14} = 0.79, x_{15} = 0.88, x_{16} = 1.36, x_{2j} = 0 (j = 1, 2, \cdots, 6)$ and the control variables are $z_4 = 1, z_5 = z_6 = 0$; $Z_{21}^0 = 4465141$ at $x_{11} = 0, x_{12} = 19.16, x_{13} = 1.28, x_{14} = 9.23, x_{15} = 9.75, x_{16} = 12.03, x_{21} = 15.71, x_{22} = 0, x_{23} = 20.31, x_{24} = 7.71, x_{25} = 8.23, x_{26} = 12.96$ and the control variables are $z_5 = 1, z_4 = z_6 = 0$.

Next, we find the linear membership functions using the equations (10) and (11) by Zimmermann method and the maximin problem for this numerical example can be written as:

$$\max \ \lambda,$$
$$\text{subject to } (Z_{11}(x) - 204.82)/(4532.97 - 204.82) \ge \lambda,$$
$$(Z_{21}(x) - 15047.83)/(4465141 - 15047.83) \ge \lambda,$$
$$x \in \mathbf{S},$$

where **S** denotes the feasible region of **Model 6**.

Using the procedure described in Sect. 4, we derive the results after the first iteration and are shown in Table 1.

Suppose that DM_{11} is not satisfied with the solution obtained in Iteration 1 and then DM_{11} specifies the minimal satisfactory level $\delta = 0.9693$ and we see that the bounds of the ratio at the interval $[\Delta_{min}, \Delta_{max}] = [0.9693, 0.9852]$, taking into account of the result of the first iteration. Then, the problem with the minimal satisfactory level is rewritten as follows:

$$\max \mu_{21}(Z_{21}(x)),$$
$$\text{subject to } (Z_{11}(x) - 204.82)/(4532.97 - 204.82) \ge 0.9693,$$
$$\text{where } x \in \mathbf{S}. \tag{15}$$

The result of the second iteration including an optimal solution to problem (12) is shown in **Table** 2 in a similar way as done in first iteration.

At the second iteration, the ratio $\Delta = 0.9999$ of satisfactory degree is not valid interval [0.9693, 0.9852] of ratio. So, DM_{11} updates the minimal satisfactory level at $\delta = 0.9777$ Then, the problem with the revised minimum satisfactory level is solved, and the result of third iteration is shown in Table 3.

Table 1 Results from iteration 1

x_{1j}:	15.71	0	0	8.13	17.98	18.76
x_{2j}:	0	19.64	21.59	8.81	0	6.23
(Z_{11}, Z_{21}):	(4399.98, 4328401)					
$(z_1, z_2, z_3, z_4, z_5, z_6)$:	(0, 1, 0, 0, 1, 0)					
$\mu_{11}(Z_{11})$:	0.9692					
$\mu_{21}(Z_{21})$:	0.9692					
λ:	0.9692					
Δ:	0.9999					

Table 2 Results from iteration 2

x_{1j}:	15.71	19.16	21.59	1.16	1	1.5
x_{2j}:	0	0	0	15.32	16.98	23.49
(Z_{11}, Z_{21}):	(4400.1, 4328283)					
$(z_1, z_2, z_3, z_4, z_5, z_6)$:	(0, 1, 0, 0, 1, 0)					
$\mu_{11}(Z_{11})$:	0.9693					
$\mu_{21}(Z_{21})$:	0.9692					
λ:	0.9692					
Δ:	0.9999					

Table 3 Results from iteration 3

x_{1j}:	15.71	19.16	21.59	3.84	1	1.5
x_{2j}:	0	0	0	13.1	16.98	23.49
(Z_{11}, Z_{21}):	(4433.42, 4294956)					
$(z_1, z_2, z_3, z_4, z_5, z_6)$:	(0, 1, 0, 0, 1, 0)					
$\mu_{11}(Z_{11})$:	0.9769					
$\mu_{21}(Z_{21})$:	0.9617					
λ:	0.9618					
Δ:	0.9844					

At the third iteration, the satisfactory degree $\mu_{11}(Z_{11}) = 0.9777$ of DM_{11} which equals to the minimum satisfactory level $\delta = 0.9777$, and the ratio $\Delta = 0.9844$ of the satisfactory degree is in the valid interval [0.9693, 0.9852] of ratio. Therefore, this solution satisfies the termination condition of the interactive process, and it becomes a compromise solution for both DMs.

Sensitivity analysis

The main intention of this study is to formulate and solve the stochastic bi-level programming problem for cooperative game in multi-choice nature under fuzzy programming technique. Let us discuss why we have considered such a study and what is the contribution of this study compared to other research works carried out by many researchers in this direction. Bi-Level Programming Problem (BLPP) has been studied by several researchers, for example, (Anandalingam 1988; Sakawa et al. 2000; Roy 2006; Lachhwani and Poonia 2012; Dey et al. 2014) and many others. Most of them have not considered when the objective functions are in multi-choice nature. Due to globalization of the market or other real-life phenomena, we have assumed that the cost parameter of the objective functions is of multi-choice type and that non-linearity occurs in the BLPP. But here we have presented the parameters of constraints that follow a normal distribution. So, our proposed method treated non-linearity when the objective functions and the constraints are both non-linear.

By conventional method, we find that the objective functions of the upper level and lower level decision makers are 4393.38 and 4465141, respectively (using LINGO 13.0 pakages) but according to our findings the results are 4433.42 and 4294956. We see that only the upper level decision maker gives a better result with respect to our proposed method. Actually, in real-life situation we always give priority to the upper level decision maker while the lower level always remains secondary. In this situation too, our proposed model works well from this point of view. Hence, taking these observations into consideration, we feel that the proposed method is a better method for our study.

Conclusion

This paper has presented the solution procedure for solving the multi-choice stochastic bi-level programming problem with consideration of normal random variable. All the probabilistic constraints have been transferred into the equivalent deterministic constraints by stochastic programming approach and a general transformation technique has used for the multi-choice cost coefficients of the objective functions using fuzzy programming technique which provides a compromise solution. From our study, it has been concluded that in a cooperative environment there exists a compromise solution which governs by the upper level decision maker.

In the real-life decision-making problem, the cost coefficients of the objective functions and the constraints may not be known previously due to uncountable factors. For this reason, the cost coefficients of the objective functions are of multi-choice rather than by single choice and the constraints are followed random variables. In this paper, we have formulated the MCSBLPP model by considering both the factors. Finally, it is obvious that the formulated model is highly applicable for these types of bi-level programming problems such as supply chain planning problem, managerial decision-making problem, facility location, transportation problem, etc. and solving this model, the decision maker has provided the optimal planning for taking the right decision.

In future study, one can extend this work, i.e., to solve the multi-choice stochastic multi-level programming problem with interval programming using fuzzy goal programming technique.

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